

## SHORT NOTES

### ELASTICITY OF SELECTED ROCKS AND MINERALS\*

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Elastic constants of selected rocks and minerals are listed, as obtained from measured values of density and of compressional and rotational wave velocities along three orthogonal directions. Density-velocity correlation and velocity-anisotropy are discussed.

#### INTRODUCTION

Elastic velocities in rocks and minerals have been the subject of numerous investigations (Birch, 1960, 1961). In this study, measurement of bulk density and of compressional and rotational wave velocities of a variety of rocks and minerals affords the opportunity of calculating elastic constants and of studying anisotropy in elastic velocities. Although it was not possible to take into account the effect of confining pressure on velocities, atmospheric-pressure values of velocities and elastic constants display considerable order in their density-dependence. This order could be increased and somewhat modified by measurement at pressures sufficiently high to reduce the effects of porosity, microfractures, and other structural defects, and by taking into account the atomic packing density, as measured by density and mean atomic weight (Birch, 1961).

#### EXPERIMENTAL METHODS

Elastic velocities were obtained by measuring the one-way transit time of an ultrasonic pulse across specimens<sup>1</sup> of cuboid shape, approximately one inch long, with opposite faces lapped parallel

within 0.002 inches, and adjacent faces perpendicular within three degrees. A thin coat of silver paint, polished after application, provided an electrical ground connection. For most of the compressional wave measurements, a 50-volt pulse of one-half  $\mu$ sec duration was applied across a two-mcps BaTiO<sub>3</sub> transducer. For rotational wave measurements, a 1,500-volt pulse<sup>2</sup> of 10- $\mu$ sec duration was applied across a two-mcps Y-cut quartz transducer. Transit times of the order of 8 and 15  $\mu$ sec for most specimens were measured to within 0.01  $\mu$ sec by conventional electronic techniques (e.g., Hughes et al, 1949).

The total error in velocity was estimated as less than  $\pm$  two percent from repeated measurement of the travel time in a given specimen. The largest source of error was the accuracy and reproducibility of identification of first arrivals. Rotational waves were identified by a sharp reduction in signal, resulting from turning one of the two transducers 90 degrees from parallel alignment of direction of polarization. The transmission path of a small precursor dilatational wave, which preceded the rotational wave in some of the specimens, was not clearly established. The transit time for a pulse between transducers

<sup>1</sup> Obtained from Wards' Natural Science Establishment.

<sup>2</sup> Arenberg Pulsed Oscillator, Model 650 C.

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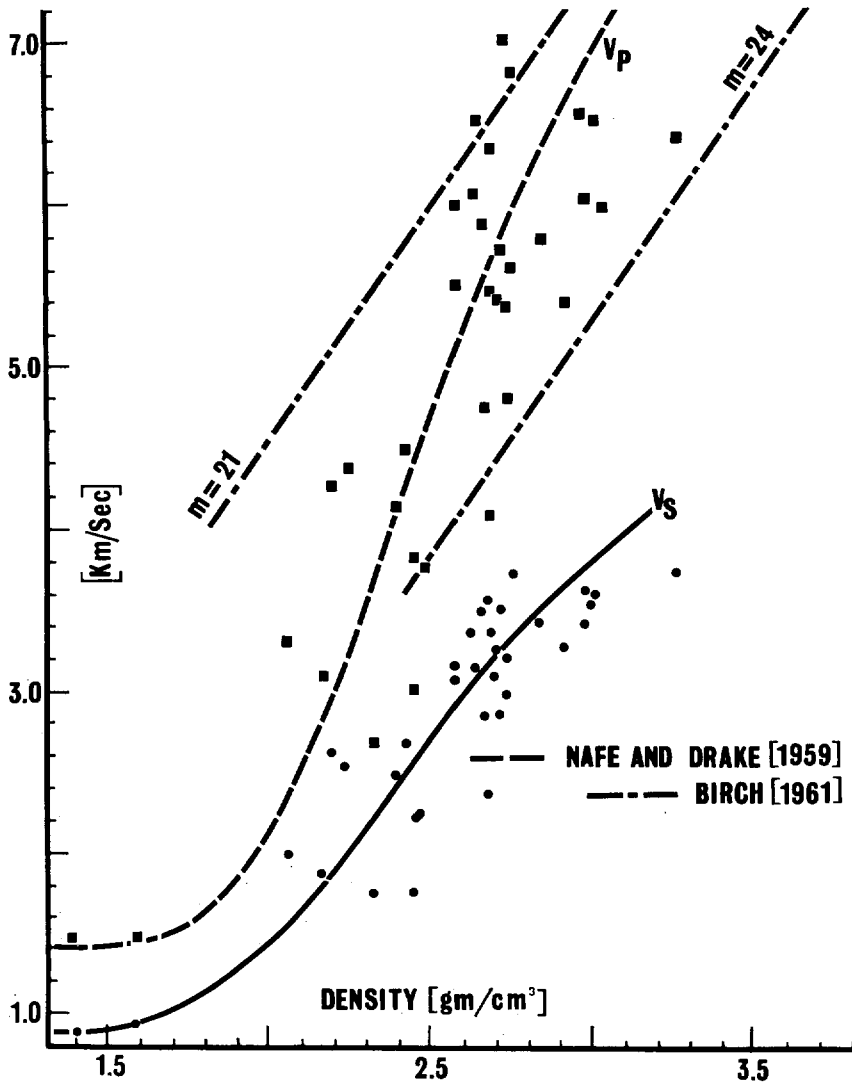


FIG. 1. Average compressional ( $V_p$ ) and rotational ( $V_s$ ) wave velocities versus density for rock specimens.

centered on opposite faces of the specimen agreed well with that of a rotational wave transformed at a lateral boundary to a compressional wave. However, the transit time was insensitive to changes in transducer position.

Hughes and Maurette (1956) have suggested an empirical correction of  $\frac{1}{4}f$ , where  $f$  is the resonant frequency of the transducer in megacycles/sec, to the travel times measured with piezoelectric crystals. For the two mcps crystals used in the present study, a correction of  $-0.125$

$\mu$  sec, approximately two percent of the transit time, is predicted by this formula. The agreement, using the above correction, between the present measurements and published values in fused silica and polycrystalline aluminum, is within a small fraction of one percent. This empirical correction, applied to all transit times, results in a velocity increase varying from two percent, for the lowest-velocity rock (tuff), to three percent for the highest-velocity rock (dunite).

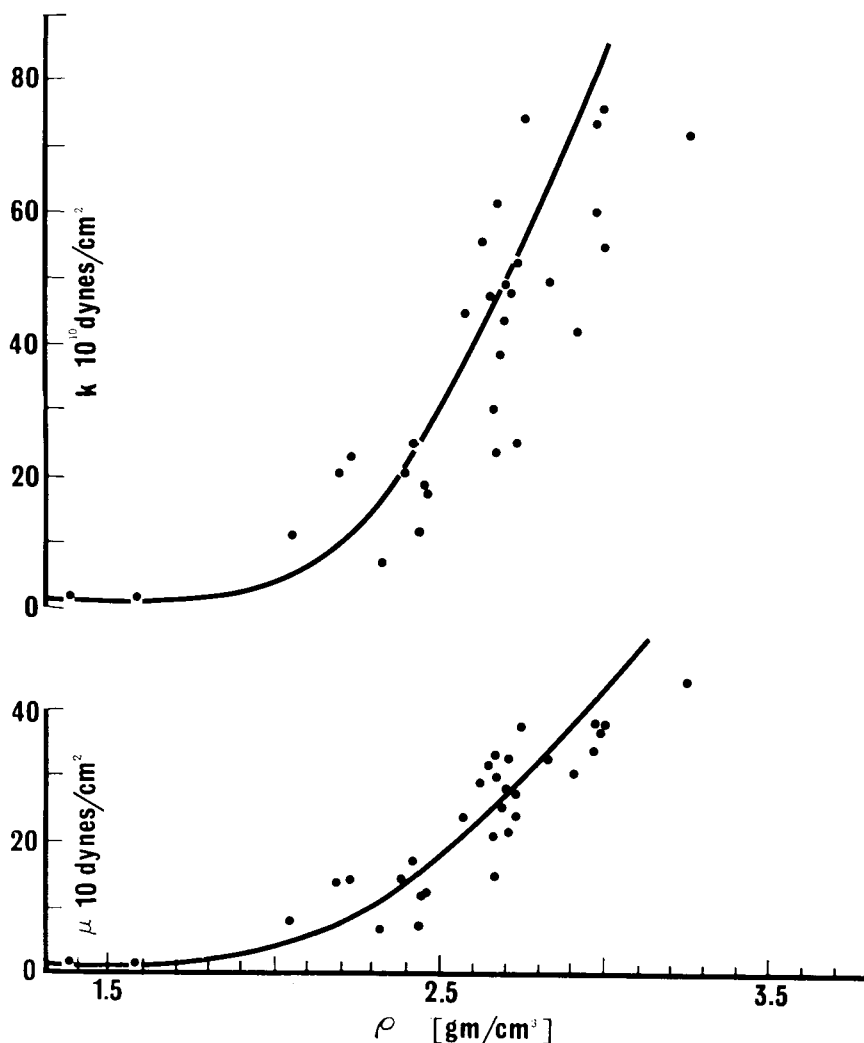


FIG. 2. Bulk modulus ( $k$ ) and rigidity ( $\mu$ ) versus density for rock specimens.

#### EXPERIMENTAL RESULTS

Corrected values of compressional ( $V_p$ ) and rotational ( $V_s$ ) wave velocities for three orthogonal directions, averages for each specimen, percentage difference between maximum and minimum values referred to the average, and calculated values of bulk modulus ( $k$ ), rigidity ( $\mu$ ), and Poisson's ratio are listed in order of density ( $\rho$ ) for rocks and minerals (Table 1). Directions of measurement, designated 1, 2, and 3 in the table, are arbitrary. In Figure 1, the average velocities

of the rock specimens are plotted, except for obsidian, which was omitted because its gross structural properties differ markedly from the other rock specimens. For comparison with compressional velocities, the empirical curve of Nafe and Drake (1959) is shown. In the absence of published information on the density-dependence of rotational velocities, an empirical curve was drawn in Figure 1, with most points falling within a range of  $\pm 10$  percent. These two empirical velocity curves (Figure 1) yield  $k$  and  $\mu$  versus  $\rho$  (Figure 2).

Table 1. Elastic wave velocities and constants.

Rock type	$\rho$ (gm/cm <sup>3</sup> )	$V_p$ (km/sec)				$\frac{\Delta V}{V_{ave}}$ (percent)	$V_s$ (km/sec)				$\frac{\Delta V'}{V_{ave}}$ (percent)	$k \times 10^{10}$ (dynes/cm <sup>2</sup> )	$\mu \times 10^{10}$ (dynes/cm <sup>2</sup> )	$\sigma$
		1	2	3	Ave.		1	2	3	Ave.				
Tuff														
San Luis Obispo, Calif.	1.38	1.41	1.36	1.51	1.43	10	0.83	0.82	0.97	0.87	17	1.40	1.06	.20
Kaolin														
Drybranch, Ga.	1.58	1.42	1.45	1.44	1.44	2	0.93	0.95	0.92	0.93	3	1.43	1.38	.13
Rhyolite														
Castle Rock, Colo.	2.05	3.47	3.19	3.16	3.27	10	1.95	2.06	1.94	1.98	6	11.2	8.07	.21
Volcanic Breccia														
Park Co., Colo.	2.19	4.26	4.19	4.21	4.22	2	2.53	2.44	2.49	2.49	4	20.9	13.6	.23
Basaltic Scoria														
Klamath Falls, Ore.	2.23	3.73	4.53	4.72	4.33	23	2.21	2.69	2.63	2.51	15	23.3	14.2	.24
Hornblende Andesite														
Mt. Shasta, Calif.	2.32	2.71	2.77	2.50	2.66	10	1.75	1.80	1.64	1.73	9	7.18	6.95	.14
Obsidian														
Lake Co., Ore.	2.35	5.85	5.82	5.80	5.82	1	3.53	3.57	3.60	3.57	2	39.8	29.9	.20
Rhyolite														
Chaffee Co., Colo.	2.39	4.21	4.05	4.04	4.10	4	2.50	2.41	2.46	2.46	4	20.9	14.5	.22
Trachytic Tuff														
Cripple Creek, Colo.	2.42	3.96	4.74	4.65	4.45	16	2.59	2.72	2.66	2.66	5	25.4	17.1	.22
Cellular Olivine Basalt														
Washington	2.44	3.10	3.00	2.88	2.99	7	1.84	1.70	1.70	1.75	8	11.9	7.45	.23
Latite														
Chaffee Co., Colo.	2.45	4.10	3.70	3.51	3.77	16	2.38	2.10	2.15	2.21	13	18.9	12.0	.23
Quartz Latite														
Chaffee Co., Colo.	2.46	3.89	3.50	3.76	3.72	10	2.23	2.14	2.32	2.23	8	17.7	12.2	.22
Vesicular Andesite														
Chaffee Co., Colo.	2.57	5.46	5.32	5.59	5.46	5	2.98	3.10	3.04	3.04	4	44.9	23.8	.27
Trachyte														
Bannockburn Twp., Ont.	2.62	6.18	5.77	6.05	6.00	7	3.38	3.21	3.38	3.32	5	55.8	28.9	.27
Biotite Muscovite														
Quartz Monzonite														
Westerly, R. I.	2.65	5.78	5.90	5.77	5.82	2	3.50	3.46	3.41	3.46	3	47.5	31.7	.23
Biotite Muscovite														
Alkali Granite														
Woodbury, Vt.	2.66	4.77	4.22	5.09	4.69	19	2.70	2.83	2.89	2.81	7	31.0	21.0	.23
Dacite														
Boulder Co., Colo.	2.67	6.20	6.33	6.36	6.30	3	3.53	3.53	3.53	3.53	0	61.6	33.3	.27
Serpentinized Peridotite														
Murfreesboro, Ark.	2.67	4.07	4.06	4.00	4.04	2	2.50	2.31	2.25	2.35	11	23.9	14.7	.24
Andesite														
Boulder Co., Colo.	2.68	5.40	5.43	5.40	5.41	1	3.34	3.38	3.27	3.33	3	38.8	29.7	.20
Basalt														
Chaffee Co., Colo.	2.69	5.32	5.41	5.37	5.37	2	3.09	3.01	3.10	3.07	3	43.8	25.3	.26
Andesite														
San Juan Co., Colo.	2.70	5.62	5.71	5.64	5.66	2	3.20	3.21	3.24	3.22	1	49.2	28.0	.26
Marble														
Tate, Ga.	2.71	5.40	5.45	5.18	5.34	5	2.84	2.79	2.87	2.83	3	48.4	21.7	.30
Basalt														
Jefferson Co., Colo.	2.73	5.55	5.58	5.54	5.56	1	2.86	2.96	3.02	2.95	5	52.7	23.8	.30
Andesite Breccia														
Ouray, Colo.	2.73	5.02	4.64	4.62	4.76	8	3.28	3.15	3.08	3.17	6	25.3	27.4	.12
Anorthosite or														
Gabbroic Anorthosite														
Duluth, Minn.	2.75	6.81	6.59	6.79	6.73	3	3.70	3.75	3.61	3.69	4	74.7	37.4	.28
Olivine Basalt														
Boulder Co., Colo.	2.83	5.71	5.83	5.67	5.74	3	3.36	3.42	3.38	3.39	2	49.9	32.5	.23
Diorite														
Jackson, Wyo.	2.91	5.54	5.38	5.10	5.34	8	3.17	3.38	3.17	3.24	6	42.3	30.5	.21

<sup>a</sup> Value used for elastic constant calculations.

Table 1. (continued)

	$\rho$ (gm /cm <sup>3</sup> )	$V_p$ (km/sec)				$\frac{\Delta V}{V_{ave}}$ (percent)	$V_s$ (km/sec)				$\frac{\Delta V}{V_{ave}}$ (percent)	$k \times 10^{10}$ (dynes /cm <sup>2</sup> )	$\mu \times 10^{10}$ (dynes /cm <sup>2</sup> )	$\sigma$
		1	2	3	Ave.		1	2	3	Ave.				
Diabase Mt. Tom, Mass.	2.97	5.97	6.00	5.90	5.96	2	3.38	3.44	3.33	3.38	3	60.1	34.0	.26
Basalt Somerset Co., N. J.	2.97	6.48	6.48	6.49	6.48	0	3.56	3.60	3.58	3.58	1	74.0	38.1	.27
Quartz Gabbro Salem, Mass.	2.99	6.27	6.70	6.42	6.46	7	3.44	3.52	3.54	3.50	3	76.2	36.6	.29
Olivine Basalt Lintz, Germany	3.00	5.93	5.93	5.91	5.92	0	3.53	3.57	3.56	3.55	1	54.7	37.8	.22
Dunite Jackson Co., N. C.	3.25	6.52	6.66	5.88	6.35	12	3.59	3.84	3.65	3.69	7	72.4	44.3	.24
<i>Mineral</i>														
Graphite Ceylon	2.16	3.30	2.93	2.94	3.06	12	1.77	1.97	1.84	1.86	11	10.3	7.47	.23
Microcline Ontario	2.57	4.75	5.57	7.49	5.94	46	2.97	3.05	3.38	3.13	13	60.2	25.2	.31
Albite Ontario	2.63	6.23	6.73	6.42	6.46	29	3.10	3.55	2.70	3.12	27	75.4	25.9	.34
Bytownite Minnesota	2.71	6.69	7.37	6.73	6.93	10	3.49	3.38	3.54	3.47	5	86.9	32.6	.33
Tremolite New York	2.86	6.25	—	6.08	6.17	3	3.83	3.02	4.25	3.70	11	55.5	39.9	.21
Diopside Quebec	3.08	3.36	5.83	7.48	5.56	74	3.11	4.07	3.96	3.71	26	46.5	43.0	.19
Augite New York	3.26	5.30 <sup>a</sup>	2.96	2.95	3.74	63	3.47 <sup>a</sup>	1.97	—	2.72	55	39.4	39.2	.13
Hornblende Ontario	3.32	7.17	5.85	6.06	6.36	21	3.05	3.14	3.79	3.33	22	85.9	37.1	.31
Limonite Alabama	3.55	5.28	5.37	5.42	5.36	3	2.95	2.96	3.00	2.97	1	60.2	31.3	.27
Pyrrhotite Ontario	4.55	4.70	4.66	4.71	4.69	1	2.78	2.71	2.78	2.76	2	53.9	34.7	.24
Pyrite Colorado	4.81	7.63	7.68	7.76	7.69	2	4.76	4.72	4.87	4.78	3	137.9	109.9	.17
Magnetite New York	4.81	4.07	4.12	4.34	4.18	7	2.09	1.57	2.25	1.97	35	58.6	19.1	.35
Hematite Michigan	4.93	6.85	7.04	6.58	6.82	8	3.91	3.84	3.78	3.84	3	132.4	72.7	.27

Birch (1960, 1961) has shown that a large improvement in the correlation of velocity and density can be achieved by measuring velocities at pressures sufficiently high to remove porosity effects and by introducing atomic weight,  $m$ , as a parameter. While petrographic analyses were performed and are available for many of the specimens, lack of chemical analyses made it impossible to determine  $m$  with precision. Nevertheless, most points of Figure 1 lie between or very near Birch's lines for  $m=21$  and  $m=24$ , a range typical of crustal rocks. Porosity effects and content of heavy metal atoms are probably responsible for most of the scatter. A plot of the velocities of the

minerals, omitted in Figures 1 and 2, shows that the points fall farther to the right the higher their atomic weight.

A number of specimens (Table 1) exhibit differences in velocities exceeding 10 percent in the three orthogonal directions. Thin-section studies of the dunite specimen ( $\rho=3.25$  gm/cm<sup>3</sup>) showed light extinction bands, indicating a preferred orientation in the space lattice within the constituent crystals.

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specimens and carrying out measurements. The counsel of J. R. Dunn, B. F. Gerhardt, and J. L. Rosenholtz is gratefully acknowledged. This research was supported in part by the Radiation Laboratory of the University of Michigan.

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## A DENSITY DETERMINATION BY UNDERGROUND GRAVITY MEASUREMENTS IN MICHIGAN\*

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The in situ density of an 1,100-ft section of Paleozoic sedimentary rocks in the southeastern portion of the Michigan Basin was determined by the established method of observing the change in gravity over a known vertical interval within the earth. The measurements were made in and adjacent to the Detroit Mine (Sec. 33, T2S, R11E) of the International Salt Company in Detroit, Michigan, which is the only deep shaft mine in the Southern Peninsula of Michigan.

Two densities were determined from gravity observations at the shafts of the mine which are 500 ft apart and a third determination was made at a vertical drill hole which intersects the mine workings approximately one mile west of the shafts. It was impossible to make gravity observations at intermediate elevations in the shafts due to the mine operations. The gravity measurements were made with an exploration gravimeter having a sensitivity of 0.01 mgal and the elevations

which were furnished by the International Salt Company are accurate to the nearest foot.

The densities were calculated from the expression for the change in the gravity ( $\Delta g$ ) over a vertical interval ( $\Delta H$ ) within the earth (Hammer, 1950),

$$\Delta g = (F - 4\pi\gamma\rho)\Delta H + \Delta t,$$

where  $F$  is the free-air vertical gradient of gravity,  $\gamma$  is the universal gravitational constant, and  $\rho$  and  $\Delta t$  are respectively the density and the variation in the terrain correction over the interval  $\Delta H$ . The use of the normal free-air vertical gradient of gravity was justified by a surface gravity survey in the vicinity of the mine which indicated no significant local gravity anomalies. This survey consisted of 46 stations which were spaced at increasing intervals from 400 ft near the mine to one mile at a distance of five miles from the mine.

Corrections due to mass effects from the surface to-

Table 1

Location	Observed gravity (mgal)	Terrain correction		Mine workings correction		Shaft correction (mgal)	Vertical interval (ft)	Density (gm/cc)
		surface	under-ground (mgal)	surface	under-ground (mgal)			
No. 1 Shaft	34.74	0	0	0.11	0.56	0.11	1,161	2.543
No. 2 Shaft	35.03	0	0	0.11	0.54	0.12	1,161	2.536
Drill hole	34.89	0	0	0.11	0.47	0.00	1,182	2.547

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